

Thinking Skills

Learn it the Feynman way, then practise it

NSW Selective High School Placement Test · for students in Year 6

7 chapters

210 questions

210 min at test pace

answers for **every** question

Each chapter starts by explaining one idea in plain words, shows you the mistake almost everyone makes, and gives you something to remember it by. Then you practise it thirty times.

Phase 0 hand-made sample. All 210 answers were independently re-derived by solver before printing (210/210 passed). Not reviewed by a qualified teacher — year-level appropriateness is unverified. Not for sale.

Chapter	Questions
1. Ordering five and six things	30 questions
2. Codes with two steps	30 questions
3. Three overlapping groups	30 questions
4. Ruling things out	30 questions
5. Days a long way out	30 questions
6. Balancing three things	30 questions
7. Machines that do two things	30 questions

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Ordering five and six things

IN PLAIN WORDS

The method does not change when the list gets longer: take one clue at a time and write down only what it forces. What changes is that you can no longer hold it all in your head.

EXAMPLE

With six runners, a clue like 'Ava finished ahead of Sam' does not tell you where either of them is. Write the six spaces down and pencil in what each clue rules out.

COMMON ERROR

Trying to do it in your head because you managed it with four. Five and six have too many arrangements. Draw the row of boxes.

REMEMBER IT LIKE THIS

It is a seating plan, not a memory game. Nobody sets a table from memory.

CHECK YOURSELF

Five children race. Mia is ahead of Sam. Sam is ahead of Ava. Is that enough to know who came first? (answer: No - two other children are still unplaced)

1. 5 children ran a race. Noah finished ahead of Finn. Noah finished somewhere between Mia and Finn. Mia finished ahead of Finn. Grace finished ahead of Noah. Mia finished immediately ahead of Noah. Finn finished somewhere between Noah and Ella. Who finished fifth?

G88MLU

☐ Finn

☐ Ella

☐ Grace

☐ Mia

working

2. 5 objects sit in a row on a shelf. the clock is between the box and the plant. the clock is somewhere left of the vase. the box is directly left of the clock. the box is between the book and the clock. the clock is somewhere left of the plant. the book is somewhere left of the box. the box is somewhere left of the plant. the vase is on the far right. What is first from the left?

REVCKL

☐ vase

☐ book

☐ box

☐ plant

working

3. 5 children ran a race. Jack finished somewhere between Noah and Grace. Noah finished immediately ahead of Jack. Jack finished ahead of Grace. Grace finished ahead of Ivy. Ivy finished immediately ahead of Hugo. Who finished fourth?

29LRMZ

☐ Hugo

☐ Jack

☐ Ivy

☐ Grace

working

4. 5 objects sit in a row on a shelf. the clock is somewhere left of the box. the plant is directly left of the lamp. the lamp is directly left of the book. the book is between the lamp and the clock. What is first from the left?

K6ETUV

☐ plant

☐ box

☐ book

☐ clock

working

5. 6 children ran a race. Zoe finished ahead of Hugo. Ella finished last. Max finished ahead of Ella. Mila finished ahead of Zoe. Leo finished ahead of Ella. Leo finished immediately ahead of Max. Zoe finished somewhere between Mila and Hugo. Zoe finished immediately ahead of Hugo. Max finished somewhere between Leo and Ella. Leo finished ahead of Max. Mila finished first. Max finished immediately ahead of Ella. Who finished fifth?

BG5JBZ

☐ Zoe

☐ Max

☐ Ella

☐ Hugo

working

6. 5 objects sit in a row on a shelf. the book is somewhere left of the bowl. the jar is on the far right. the vase is somewhere left of the jar. the book is directly left of the bowl. the plant is between the vase and the jar. the vase is between the bowl and the plant. What is third from the left?

M66GZE

☐ jar

☐ plant

☐ vase

☐ book

working

7. 6 children ran a race. Eli finished ahead of Kai. Eli finished immediately ahead of Sam. Sam finished somewhere between Eli and Kai. Kai finished ahead of Grace. Sam finished ahead of Grace. Ella finished ahead of Sam. Finn finished ahead of Ella. Who finished fourth?

X8UNQF

☐ Finn

☐ Eli

☐ Sam

☐ Ella

working

8. 5 objects sit in a row on a shelf. the plant is directly left of the vase. the plant is somewhere left of the vase. the book is somewhere left of the plant. the vase is on the far right. the box is somewhere left of the jar. the plant is between the jar and the vase. the book is directly left of the jar. What is first from the left?

LBYA8K

☐ jar

☐ plant

☐ vase

☐ box

working

9. 5 children ran a race. Ivy finished immediately ahead of Eli. Grace finished ahead of Ivy. Ivy finished ahead of Eli. Grace finished immediately ahead of Ava. Ivy finished ahead of Mia. Who finished fourth?

4A2DPR

☐ Eli

☐ Grace

☐ Ava

☐ Ivy

working

10. 5 objects sit in a row on a shelf. the plant is somewhere left of the bowl. the lamp is somewhere left of the jar. the plant is directly left of the bowl. the clock is on the far right. the bowl is directly left of the clock. What is first from the left?

TTE9AE

☐ bowl

☐ lamp

☐ clock

☐ plant

working

11. 5 children ran a race. Grace finished immediately ahead of Lily. Finn finished immediately ahead of Grace. Sam finished immediately ahead of Finn. Lily finished immediately ahead of Kai. Who finished first?

QX5QG5

☐ Finn

☐ Sam

☐ Grace

☐ Lily

working

12. 5 objects sit in a row on a shelf. the box is between the jar and the lamp. the jar is directly left of the box. the box is somewhere left of the lamp. the lamp is somewhere left of the vase. the vase is directly left of the plant. What is fourth from the left?

B4BDWM

☐ plant

☐ lamp

☐ box

☐ vase

working

13. 6 children ran a race. Finn finished ahead of Max. Max finished ahead of Jack. Kai finished immediately ahead of Jack. Finn finished immediately ahead of Max. Max finished somewhere between Finn and Kai. Ivy finished immediately ahead of Finn. Ivy finished somewhere between Eli and Finn. Who finished fourth?

VRA2UK

☐ Kai

☐ Eli

☐ Max

☐ Jack

working

14. 5 objects sit in a row on a shelf. the clock is somewhere left of the plant. the clock is somewhere left of the jar. the plant is between the jar and the book. the book is directly left of the vase. the plant is somewhere left of the book. What is third from the left? NPAE68

☐ jar

☐ book

☐ vase

☐ plant

working

15. 5 children ran a race. Sam finished somewhere between Kai and Grace. Ava finished ahead of Sam. Sam finished ahead of Grace. Ruby finished ahead of Ava. Grace finished last. Ruby finished first. Kai finished somewhere between Ava and Sam. Who finished second? 2EA838

☐ Ruby

☐ Kai

☐ Sam

☐ Ava

working

16. 5 objects sit in a row on a shelf. the box is somewhere left of the jar. the clock is directly left of the box. the box is directly left of the jar. the clock is somewhere left of the jar. the bowl is directly left of the plant. the jar is somewhere left of the plant. What is third from the left? HFWD6F

☐ bowl

☐ jar

☐ clock

☐ box

working

17. 6 children ran a race. Max finished somewhere between Owen and Ava. Owen finished ahead of Max. Noah finished somewhere between Finn and Eli. Owen finished immediately ahead of Max. Finn finished immediately ahead of Noah. Finn finished ahead of Eli. Eli finished last. Max finished ahead of Finn. Noah finished ahead of Eli. Finn finished somewhere between Ava and Noah. Who finished first? Z6SP2Z

☐ Noah

☐ Ava

☐ Owen

☐ Finn

working

18. 5 objects sit in a row on a shelf. the lamp is somewhere left of the book. the box is on the far right. the lamp is somewhere left of the vase. the lamp is directly left of the book. the book is somewhere left of the clock. the clock is between the vase and the box. What is third from the left? X6H9WC

☐ box

☐ lamp

☐ vase

☐ book

working

19. 6 children ran a race. Max finished somewhere between Ruby and Grace. Eli finished last. Grace finished ahead of Mila. Grace finished immediately ahead of Mila. Ruby finished somewhere between Leo and Max. Leo finished ahead of Ruby. Who finished fifth?

HY57Q8

☐ Leo

☐ Ruby

☐ Mila

☐ Max

working

20. 5 objects sit in a row on a shelf. the bowl is directly left of the box. the clock is somewhere left of the bowl. the clock is on the far left. the box is between the bowl and the lamp. the clock is directly left of the vase. What is fourth from the left?

9ENAN6

☐ box

☐ clock

☐ vase

☐ bowl

working

21. 5 children ran a race. Nina finished somewhere between Finn and Ella. Nina finished immediately ahead of Ella. Finn finished immediately ahead of Nina. Finn finished somewhere between Owen and Nina. Finn finished ahead of Ella. Nina finished ahead of Ella. Owen finished immediately ahead of Finn. Hugo finished last. Who finished fourth?

Q3EJJK

☐ Finn

☐ Hugo

☐ Owen

☐ Ella

working

22. 5 objects sit in a row on a shelf. the plant is somewhere left of the book. the box is on the far right. the plant is somewhere left of the jar. the vase is somewhere left of the box. the jar is somewhere left of the vase. the plant is directly left of the jar. the book is somewhere left of the vase. What is first from the left?

ZZGRMQ

☐ jar

☐ vase

☐ plant

☐ box

working

23. 6 children ran a race. Hugo finished immediately ahead of Mila. Ava finished somewhere between Ella and Kai. Grace finished ahead of Hugo. Grace finished immediately ahead of Hugo. Mila finished last. Kai finished ahead of Grace. Ava finished immediately ahead of Kai. Who finished fifth?

UNEW99

☐ Ella

☐ Mila

☐ Hugo

☐ Grace

working

24. 5 objects sit in a row on a shelf. the clock is somewhere left of the lamp. the lamp is directly left of the vase. the book is somewhere left of the vase. the book is between the clock and the lamp. the clock is directly left of the book. the book is directly left of the lamp. the box is directly left of the clock. What is fourth from the left? 4K4L4V

☐ clock

☐ box

☐ lamp

☐ vase

working

25. 5 children ran a race. Sam finished ahead of Noah. Finn finished ahead of Noah. Sam finished somewhere between Ivy and Finn. Zoe finished ahead of Ivy. Sam finished ahead of Finn. Who finished fifth? C4TLKL

☐ Ivy

☐ Sam

☐ Noah

☐ Finn

working

26. 5 objects sit in a row on a shelf. the box is somewhere left of the lamp. the box is somewhere left of the plant. the jar is somewhere left of the plant. the clock is on the far right. the plant is directly left of the lamp. the lamp is somewhere left of the clock. the box is directly left of the plant. What is third from the left? ZW34FX

☐ plant

☐ clock

☐ box

☐ jar

working

27. 5 children ran a race. Zoe finished last. Max finished somewhere between Sam and Ivy. Mila finished ahead of Sam. Ivy finished immediately ahead of Zoe. Who finished fourth? SL8H6X

☐ Sam

☐ Mila

☐ Zoe

☐ Ivy

working

28. 5 objects sit in a row on a shelf. the clock is somewhere left of the box. the vase is somewhere left of the jar. the box is somewhere left of the plant. the jar is directly left of the box. the box is between the jar and the plant. the plant is on the far right. the clock is between the vase and the jar. What is fourth from the left? YKCLVW

☐ vase

☐ clock

☐ box

☐ plant

working

29. 6 children ran a race. Sam finished ahead of Kai. Kai finished ahead of Owen. Sam finished ahead of Leo. Jack finished first. Jack finished immediately ahead of Sam. Owen finished somewhere between Kai and Max. Owen finished immediately ahead of Max. Leo finished somewhere between Sam and Kai. Who finished fifth?

6CE86Z

☐ Jack

☐ Owen

☐ Leo

☐ Max

working

30. 5 objects sit in a row on a shelf. the bowl is on the far left. the lamp is between the vase and the book. the vase is directly left of the lamp. the book is between the lamp and the box. What is fourth from the left?

53CALP

☐ book

☐ bowl

☐ box

☐ lamp

working

Chapter 1 — Answers: Ordering five and six things

1. Ella

Put them in order from the clues: Grace then Mia then Noah then Finn then Ella. So fifth is Ella.

2. book

From the clues the row is: book, box, clock, plant, vase. first from the left is the book.

3. Ivy

Put them in order from the clues: Noah then Jack then Grace then Ivy then Hugo. So fourth is Ivy.

4. plant

From the clues the row is: plant, lamp, book, clock, box. first from the left is the plant.

5. Max

Put them in order from the clues: Mila then Zoe then Hugo then Leo then Max then Ella. So fifth is Max.

6. vase

From the clues the row is: book, bowl, vase, plant, jar. third from the left is the vase.

7. Sam

Put them in order from the clues: Finn then Ella then Eli then Sam then Kai then Grace. So fourth is Sam.

8. box

From the clues the row is: box, book, jar, plant, vase. first from the left is the box.

9. Eli

Put them in order from the clues: Grace then Ava then Ivy then Eli then Mia. So fourth is Eli.

10. lamp

From the clues the row is: lamp, jar, plant, bowl, clock. first from the left is the lamp.

11. Sam

Put them in order from the clues: Sam then Finn then Grace then Lily then Kai. So first is Sam.

12. vase

From the clues the row is: jar, box, lamp, vase, plant. fourth from the left is the vase.

13. Max

Put them in order from the clues: Eli then Ivy then Finn then Max then Kai then Jack. So fourth is Max.

14. plant

From the clues the row is: clock, jar, plant, book, vase. third from the left is the plant.

15. Ava

Put them in order from the clues: Ruby then Ava then Kai then Sam then Grace. So second is Ava.

16. jar

From the clues the row is: clock, box, jar, bowl, plant. third from the left is the jar.

17. Owen

Put them in order from the clues: Owen then Max then Ava then Finn then Noah then Eli. So first is Owen.

18. vase

From the clues the row is: lamp, book, vase, clock, box. third from the left is the vase.

19. Mila

Put them in order from the clues: Leo then Ruby then Max then Grace then Mila then Eli. So fifth is Mila.

20. box

From the clues the row is: clock, vase, bowl, box, lamp. fourth from the left is the box.

21. Ella

Put them in order from the clues: Owen then Finn then Nina then Ella then Hugo. So fourth is Ella.

22. plant

From the clues the row is: plant, jar, book, vase, box. first from the left is the plant.

23. Hugo

Put them in order from the clues: Ella then Ava then Kai then Grace then Hugo then Mila. So fifth is Hugo.

24. lamp

From the clues the row is: box, clock, book, lamp, vase. fourth from the left is the lamp.

25. Noah

Put them in order from the clues: Zoe then Ivy then Sam then Finn then Noah. So fifth is Noah.

26. plant

From the clues the row is: jar, box, plant, lamp, clock. third from the left is the plant.

27. Ivy

Put them in order from the clues: Mila then Sam then Max then Ivy then Zoe. So fourth is Ivy.

28. box

From the clues the row is: vase, clock, jar, box, plant. fourth from the left is the box.

29. Owen

Put them in order from the clues: Jack then Sam then Leo then Kai then Owen then Max. So fifth is Owen.

30. book

From the clues the row is: bowl, vase, lamp, book, box. fourth from the left is the book.

Codes with two steps

IN PLAIN WORDS

A two-step code does one thing, then does another thing to the RESULT. The second step never looks at the original word again.

EXAMPLE

Shift cat forward by one and you get dbu. Then reverse it: ubd. Not tac reversed, not cat reversed - the reverse happens to dbu.

COMMON ERROR

Applying the second step to the original word. If you reverse cat first you get tac, and everything after that is wrong.

REMEMBER IT LIKE THIS

A production line. The second machine only ever sees what the first machine handed it.

CHECK YOURSELF

Shift dog forward by two, then reverse. What is the first step? (answer: fqi - do the shift before anything else)

1. A code moves every letter 1 place forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for lamb? GMBH5V

☐ bmal

☐ mmbnc

☐ mbnc

☐ kzla

working

2. A code moves every letter 2 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for wolf? 3CN9HU

☐ umjd

☐ flow

☐ yqnh

☐ yyqnh

working

3. A code moves every letter 3 places forward in the alphabet, and then you write the letters backwards. What is the code for dog? 4VCAF2

☐ grj

☐ jrg

☐ ald

☐ god

working

4. A code moves every letter 2 places forward in the alphabet, and then you write the letters backwards. What is the code for dog?

RMJZLW

☐ god

☐ bme

☐ iqf

☐ fqi

working

5. A code moves every letter 2 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for cat?

MUS332

☐ ecv

☐ ayr

☐ eecv

☐ tac

working

6. A code moves every letter 5 places forward in the alphabet, and then you write the letters backwards. What is the code for lamb?

YYVEGB

☐ qfrg

☐ bmal

☐ grfq

☐ gvhw

working

7. A code moves every letter 2 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for bird?

FNTQA9

☐ drib

☐ zgp b

☐ dktf

☐ ddktf

working

8. A code moves every letter 3 places forward in the alphabet, and then you write the letters backwards. What is the code for wolf?

DX2KS7

☐ flow

☐ tlic

☐ zroi

☐ iorz

working

9. A code moves every letter 5 places forward in the alphabet, and then you write the letters backwards. What is the code for goat?

M9RDJL

☐ bjvo

☐ lt fy

☐ taog

☐ yftl

working

10. A code moves every letter 1 place forward in the alphabet, and then you write the letters backwards. JRRMXD
What is the code for dog?

☐ hpe ☐ cnf ☐ god ☐ eph

working

11. A code moves every letter 2 places forward in the alphabet, and then you write the letters backwards. 4N5M45
What is the code for goat?

☐ vcqi ☐ emyr ☐ taog ☐ iqcv

working

12. A code moves every letter 4 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for cat? EHE9EY

☐ tac ☐ ggex ☐ ywp ☐ gex

working

13. A code moves every letter 4 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for goat? JG27BN

☐ ksex ☐ ckwp ☐ taog ☐ kksex

working

14. A code moves every letter 2 places forward in the alphabet, and then you write the letters backwards. RLLMZS
What is the code for fish?

☐ jukh ☐ dgqf ☐ hkuj ☐ hsif

working

15. A code moves every letter 3 places forward in the alphabet, and then you write the letters backwards. LWKNQP
What is the code for lamb?

☐ bmal ☐ ixjy ☐ odpe ☐ epdo

working

16. A code moves every letter 2 places forward in the alphabet, and then you write the letters backwards. HXJQB9
What is the code for lamb?

- ☐ ncod ☐ docn ☐ bmal ☐ jyz

working

17. A code moves every letter 2 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for dog? KMK45P

- ☐ bme ☐ god ☐ ffqi ☐ fqi

working

18. A code moves every letter 4 places forward in the alphabet, and then you write the letters backwards. MSZRUB
What is the code for frog?

- ☐ ksvj ☐ gorf ☐ bnkc ☐ jvsk

working

19. A code moves every letter 1 place forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for duck? 9YD2BB

- ☐ eevdl ☐ kcud ☐ ctbj ☐ evdl

working

20. A code moves every letter 5 places forward in the alphabet, and then you write the letters backwards. SGKA4M
What is the code for fish?

- ☐ adnc ☐ mxnk ☐ knxm ☐ hsif

working

21. A code moves every letter 5 places forward in the alphabet, and then you write the letters backwards. K7MWJJ
What is the code for bird?

- ☐ iwng ☐ drib ☐ wdmy ☐ gnwi

working

22. A code moves every letter 4 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for duck?

SWMQAN

☐ hygo

☐ hhygo

☐ zqyg

☐ kcud

working

23. A code moves every letter 4 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for fish?

CA3C2C

☐ beod

☐ jmwI

☐ jjmwI

☐ hsif

working

24. A code moves every letter 3 places forward in the alphabet, and then you write the letters backwards. What is the code for duck?

22TZKT

☐ gxfn

☐ nfxg

☐ kcud

☐ arzh

working

25. A code moves every letter 5 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for cat?

ABZ3KV

☐ hfy

☐ xvo

☐ tac

☐ hhfy

working

26. A code moves every letter 3 places forward in the alphabet, and then you write the letters backwards. What is the code for frog?

P8S54C

☐ iurj

☐ jrui

☐ gorf

☐ cold

working

27. A code moves every letter 4 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for bird?

GS3B9M

☐ drib

☐ ffmvh

☐ xenz

☐ fmvh

working

28. A code moves every letter 4 places forward in the alphabet, and then you write the letters backwards. KLVGTM
What is the code for bird?

☐ drib

☐ hvmf

☐ fmvh

☐ xenz

working

29. A code moves every letter 1 place forward in the alphabet, and then you write the letters backwards. R4G6VJ
What is the code for cat?

☐ dbu

☐ ubd

☐ tac

☐ bzs

working

30. A code moves every letter 3 places forward in the alphabet, and then you write the first letter of the code twice at the start. What is the code for frog? K7344M

☐ iiurj

☐ gorf

☐ cold

☐ iurj

working

Chapter 2 — Answers: Codes with two steps

1. **mmbnc**

Move each letter 1 forward: lamb becomes mbnc. Then write the first letter of the code twice at the start: mmbnc.

2. **yyqnh**

Move each letter 2 forward: wolf becomes yqnh. Then write the first letter of the code twice at the start: yyqnh.

3. **jrg**

Move each letter 3 forward: dog becomes grj. Then write the letters backwards: jrg.

4. **iqf**

Move each letter 2 forward: dog becomes fqi. Then write the letters backwards: iqf.

5. **eecv**

Move each letter 2 forward: cat becomes ecv. Then write the first letter of the code twice at the start: eecv.

6. **grfq**

Move each letter 5 forward: lamb becomes qfrg. Then write the letters backwards: grfq.

7. **ddktf**

Move each letter 2 forward: bird becomes dktf. Then write the first letter of the code twice at the start: ddktf.

8. **iorz**

Move each letter 3 forward: wolf becomes zroi. Then write the letters backwards: iorz.

9. **yftl**

Move each letter 5 forward: goat becomes ltfy. Then write the letters backwards: yftl.

10. **hpe**

Move each letter 1 forward: dog becomes eph. Then write the letters backwards: hpe.

11. **vcqi**

Move each letter 2 forward: goat becomes iqcv. Then write the letters backwards: vcqi.

12. **ggex**

Move each letter 4 forward: cat becomes gex. Then write the first letter of the code twice at the start: ggex.

13. **kksex**

Move each letter 4 forward: goat becomes ksex. Then write the first letter of the code twice at the start: kksex.

14. **jukh**

Move each letter 2 forward: fish becomes hkuj. Then write the letters backwards: jukh.

15. **epdo**

Move each letter 3 forward: lamb becomes odpe. Then write the letters backwards: epdo.

16. **docn**

Move each letter 2 forward: lamb becomes ncod. Then write the letters backwards: docn.

17. **ffqi**

Move each letter 2 forward: dog becomes fqi. Then write the first letter of the code twice at the start: ffqi.

18. **ksvj**

Move each letter 4 forward: frog becomes jvsk. Then write the letters backwards: ksvj.

19. **eevdl**

Move each letter 1 forward: duck becomes evdl. Then write the first letter of the code twice at the start: eevdl.

20. **mxnk**

Move each letter 5 forward: fish becomes knxm. Then write the letters backwards: mxnk.

21. **iwng**

Move each letter 5 forward: bird becomes gnwi. Then write the letters backwards: iwng.

22. hhygo

Move each letter 4 forward: duck becomes hygo. Then write the first letter of the code twice at the start: hhygo.

23. jjmwl

Move each letter 4 forward: fish becomes jmwL. Then write the first letter of the code twice at the start: jjmwl.

24. nfxg

Move each letter 3 forward: duck becomes gxfn. Then write the letters backwards: nfxg.

25. hhfy

Move each letter 5 forward: cat becomes hfy. Then write the first letter of the code twice at the start: hhfy.

26. jrui

Move each letter 3 forward: frog becomes iurj. Then write the letters backwards: jrui.

27. ffmvh

Move each letter 4 forward: bird becomes fmvh. Then write the first letter of the code twice at the start: ffmvh.

28. hvmf

Move each letter 4 forward: bird becomes fmvh. Then write the letters backwards: hvmf.

29. ubd

Move each letter 1 forward: cat becomes dbu. Then write the letters backwards: ubd.

30. iiurj

Move each letter 3 forward: frog becomes iurj. Then write the first letter of the code twice at the start: iiurj.

Three overlapping groups

IN PLAIN WORDS

When groups overlap, someone in two groups gets counted twice unless you take the overlap off. With three groups the middle piece gets counted three times, so it needs more care, not more panic.

EXAMPLE

12 own a dog, 9 own a cat, 4 own both. Dog or cat is $12 + 9 - 4 = 17$, not 21. The four are in both counts already.

COMMON ERROR

Adding the group sizes and stopping. The question usually gives you the overlap precisely because you are meant to subtract it.

REMEMBER IT LIKE THIS

Two guest lists for one party. Add them up and the people invited twice get two chairs each.

CHECK YOURSELF

10 play tennis, 8 play netball, 3 play both. How many play at least one? (answer: 15)

1. In a class of 29 students, everyone plays at least one of three sports. 9 play only tennis, 2 play only netball, 9 play only hockey, 2 play tennis and netball but not hockey, 3 play tennis and hockey but not netball, 2 play netball and hockey but not tennis, and 2 play all three. How many play tennis?

25L7DB

☐ 9☐ 18☐ 16☐ 14

working

2. In a group of 26 people, 12 own a dog and 13 own a cat. 4 own both. How many own neither?

87H4GH

☐ 4☐ 17☐ 22☐ 5

working

3. In a class of 28 students, everyone plays at least one of three sports. 6 play only tennis, 7 play only netball, 5 play only hockey, 5 play tennis and netball but not hockey, 1 play tennis and hockey but not netball, 3 play netball and hockey but not tennis, and 1 play all three. How many play tennis?

57EGD7

☐ 13☐ 12☐ 6☐ 16

working

4. In a group of 25 people, 12 own a dog and 10 own a cat. 2 own both. How many own neither?

ZLGUE5

☐ 23

☐ 18

☐ 5

☐ 2

working

5. In a class of 30 students, everyone plays at least one of three sports. 8 play only tennis, 8 play only netball, 5 play only hockey, 1 play tennis and netball but not hockey, 1 play tennis and hockey but not netball, 3 play netball and hockey but not tennis, and 4 play all three. How many play tennis?

ULL56P

☐ 10

☐ 8

☐ 17

☐ 14

working

6. In a group of 21 people, 9 own a dog and 11 own a cat. 5 own both. How many own neither?

H2A2BU

☐ 6

☐ 10

☐ 5

☐ 16

working

7. In a class of 31 students, everyone plays at least one of three sports. 9 play only tennis, 3 play only netball, 8 play only hockey, 4 play tennis and netball but not hockey, 1 play tennis and hockey but not netball, 3 play netball and hockey but not tennis, and 3 play all three. How many play tennis?

44E7AF

☐ 17

☐ 20

☐ 14

☐ 9

working

8. In a group of 17 people, 9 own a dog and 9 own a cat. 5 own both. How many own neither?

DBKA63

☐ 4

☐ 5

☐ 12

☐ 8

working

9. In a class of 34 students, everyone plays at least one of three sports. 3 play only tennis, 9 play only netball, 8 play only hockey, 5 play tennis and netball but not hockey, 1 play tennis and hockey but not netball, 4 play netball and hockey but not tennis, and 4 play all three. How many play tennis?

DU7J47

☐ 3

☐ 17

☐ 9

☐ 13

working

10. In a group of 19 people, 12 own a dog and 5 own a cat. 2 own both. How many own neither?

BMJ8EF

☐ 17

☐ 2

☐ 4

☐ 13

working

11. In a class of 33 students, everyone plays at least one of three sports. 5 play only tennis, 4 play only netball, 6 play only hockey, 5 play tennis and netball but not hockey, 5 play tennis and hockey but not netball, 5 play netball and hockey but not tennis, and 3 play all three. How many play tennis?

W5UPN3

☐ 23

☐ 15

☐ 5

☐ 18

working

12. In a group of 24 people, 9 own a dog and 16 own a cat. 6 own both. How many own neither?

UTWYQD

☐ 6

☐ 18

☐ 5

☐ 13

working

13. In a class of 24 students, everyone plays at least one of three sports. 5 play only tennis, 4 play only netball, 3 play only hockey, 4 play tennis and netball but not hockey, 2 play tennis and hockey but not netball, 4 play netball and hockey but not tennis, and 2 play all three. How many play tennis?

ANG6U7

☐ 17

☐ 5

☐ 11

☐ 13

working

14. In a group of 23 people, 9 own a dog and 15 own a cat. 6 own both. How many own neither?

NDNDBW

☐ 12

☐ 6

☐ 5

☐ 17

working

15. In a class of 33 students, everyone plays at least one of three sports. 9 play only tennis, 5 play only netball, 7 play only hockey, 1 play tennis and netball but not hockey, 2 play tennis and hockey but not netball, 5 play netball and hockey but not tennis, and 4 play all three. How many play tennis?

ZKC554

☐ 9

☐ 21

☐ 12

☐ 16

working

16. In a group of 21 people, 9 own a dog and 11 own a cat. 2 own both. How many own neither?

GADZU9

☐ 3

☐ 16

☐ 2

☐ 19

working

17. In a class of 17 students, everyone plays at least one of three sports. 4 play only tennis, 2 play only netball, 3 play only hockey, 1 play tennis and netball but not hockey, 2 play tennis and hockey but not netball, 1 play netball and hockey but not tennis, and 4 play all three. How many play tennis?

8YED5A

☐ 11

☐ 7

☐ 4

☐ 12

working

18. In a group of 15 people, 7 own a dog and 5 own a cat. 2 own both. How many own neither?

XU9YWN

☐ 13

☐ 8

☐ 2

☐ 5

working

19. In a class of 33 students, everyone plays at least one of three sports. 6 play only tennis, 9 play only netball, 3 play only hockey, 1 play tennis and netball but not hockey, 5 play tennis and hockey but not netball, 5 play netball and hockey but not tennis, and 4 play all three. How many play tennis?

D8JCXJ

☐ 6

☐ 21

☐ 12

☐ 16

working

20. In a group of 21 people, 12 own a dog and 12 own a cat. 5 own both. How many own neither?

XBQVRH

☐ 2

☐ 14

☐ 5

☐ 16

working

21. In a class of 24 students, everyone plays at least one of three sports. 9 play only tennis, 5 play only netball, 4 play only hockey, 2 play tennis and netball but not hockey, 1 play tennis and hockey but not netball, 2 play netball and hockey but not tennis, and 1 play all three. How many play tennis?

3B773T

☐ 9

☐ 15

☐ 13

☐ 12

working

22. In a group of 23 people, 9 own a dog and 10 own a cat. 3 own both. How many own neither?

6U7MFR

☐ 13

☐ 3

☐ 20

☐ 7

working

23. In a class of 30 students, everyone plays at least one of three sports. 7 play only tennis, 8 play only netball, 3 play only hockey, 5 play tennis and netball but not hockey, 1 play tennis and hockey but not netball, 3 play netball and hockey but not tennis, and 3 play all three. How many play tennis?

KBLA8J

☐ 7

☐ 16

☐ 13

☐ 19

working

24. In a group of 21 people, 10 own a dog and 9 own a cat. 6 own both. How many own neither?

AAZVWB

☐ 15

☐ 7

☐ 8

☐ 6

working

25. In a class of 20 students, everyone plays at least one of three sports. 2 play only tennis, 6 play only netball, 6 play only hockey, 1 play tennis and netball but not hockey, 2 play tennis and hockey but not netball, 2 play netball and hockey but not tennis, and 1 play all three. How many play tennis?

RJHCKB

☐ 2

☐ 8

☐ 6

☐ 5

working

26. In a group of 26 people, 12 own a dog and 13 own a cat. 6 own both. How many own neither?

4NJYSQ

☐ 13

☐ 6

☐ 20

☐ 7

working

27. In a class of 29 students, everyone plays at least one of three sports. 8 play only tennis, 6 play only netball, 5 play only hockey, 2 play tennis and netball but not hockey, 4 play tennis and hockey but not netball, 2 play netball and hockey but not tennis, and 2 play all three. How many play tennis?

CPHZB8

☐ 16

☐ 14

☐ 18

☐ 8

working

28. In a group of 21 people, 12 own a dog and 11 own a cat. 5 own both. How many own neither?

8YQSTZ

☐ 5

☐ 13

☐ 16

☐ 3

working

29. In a class of 33 students, everyone plays at least one of three sports. 8 play only tennis, 2 play only netball, 8 play only hockey, 3 play tennis and netball but not hockey, 4 play tennis and hockey but not netball, 4 play netball and hockey but not tennis, and 4 play all three. How many play tennis?

VLE5K9

☐ 15

☐ 23

☐ 19

☐ 8

working

30. In a group of 26 people, 16 own a dog and 12 own a cat. 6 own both. How many own neither?

2T3SF6

☐ 4

☐ 16

☐ 20

☐ 6

working

Chapter 3 — Answers: Three overlapping groups

1. 16

Tennis is everyone in any tennis region: $9 + 2 + 3 + 2 = 16$.

2. 5

Dog or cat is $12 + 13 - 4 = 21$. Neither is $26 - 21 = 5$.

3. 13

Tennis is everyone in any tennis region: $6 + 5 + 1 + 1 = 13$.

4. 5

Dog or cat is $12 + 10 - 2 = 20$. Neither is $25 - 20 = 5$.

5. 14

Tennis is everyone in any tennis region: $8 + 1 + 1 + 4 = 14$.

6. 6

Dog or cat is $9 + 11 - 5 = 15$. Neither is $21 - 15 = 6$.

7. 17

Tennis is everyone in any tennis region: $9 + 4 + 1 + 3 = 17$.

8. 4

Dog or cat is $9 + 9 - 5 = 13$. Neither is $17 - 13 = 4$.

9. 13

Tennis is everyone in any tennis region: $3 + 5 + 1 + 4 = 13$.

10. 4

Dog or cat is $12 + 5 - 2 = 15$. Neither is $19 - 15 = 4$.

11. 18

Tennis is everyone in any tennis region: $5 + 5 + 5 + 3 = 18$.

12. 5

Dog or cat is $9 + 16 - 6 = 19$. Neither is $24 - 19 = 5$.

13. 13

Tennis is everyone in any tennis region: $5 + 4 + 2 + 2 = 13$.

14. 5

Dog or cat is $9 + 15 - 6 = 18$. Neither is $23 - 18 = 5$.

15. 16

Tennis is everyone in any tennis region: $9 + 1 + 2 + 4 = 16$.

16. 3

Dog or cat is $9 + 11 - 2 = 18$. Neither is $21 - 18 = 3$.

17. 11

Tennis is everyone in any tennis region: $4 + 1 + 2 + 4 = 11$.

18. 5

Dog or cat is $7 + 5 - 2 = 10$. Neither is $15 - 10 = 5$.

19. 16

Tennis is everyone in any tennis region: $6 + 1 + 5 + 4 = 16$.

20. 2

Dog or cat is $12 + 12 - 5 = 19$. Neither is $21 - 19 = 2$.

21. 13

Tennis is everyone in any tennis region: $9 + 2 + 1 + 1 = 13$.

22. 7

Dog or cat is $9 + 10 - 3 = 16$. Neither is $23 - 16 = 7$.

23. 16

Tennis is everyone in any tennis region: $7 + 5 + 1 + 3 = 16$.

24. 8

Dog or cat is $10 + 9 - 6 = 13$. Neither is $21 - 13 = 8$.

25. 6

Tennis is everyone in any tennis region: $2 + 1 + 2 + 1 = 6$.

26. 7

Dog or cat is $12 + 13 - 6 = 19$. Neither is $26 - 19 = 7$.

27. 16

Tennis is everyone in any tennis region: $8 + 2 + 4 + 2 = 16$.

28. 3

Dog or cat is $12 + 11 - 5 = 18$. Neither is $21 - 18 = 3$.

29. 19

Tennis is everyone in any tennis region: $8 + 3 + 4 + 4 = 19$.

30. 4

Dog or cat is $16 + 12 - 6 = 22$. Neither is $26 - 22 = 4$.

Ruling things out

IN PLAIN WORDS

If every item belongs to exactly one person and you are told all but one, the last one is settled. You do not need a clue about it.

EXAMPLE

Five children, five different pets, and four children named with their pet. The fifth child owns whichever pet is left.

COMMON ERROR

Looking for a clue that names the person you were asked about. There is not one, and there does not need to be.

REMEMBER IT LIKE THIS

Five coats on five hooks. Four people take theirs; the last coat is yours whether anyone said so or not.

CHECK YOURSELF

Four children own a cat, dog, bird and fish. Three are named. Do you need a fourth clue? (answer: No - the remaining pet belongs to the remaining child)

1. 5 children each own a different pet: a cat, a fish, a rabbit, a dog and a bird. Hugo owns a cat. Grace owns a rabbit. Sam owns a bird. Noah owns a dog. What does Mia own? J6FQPB

☐ a rabbit

☐ a fish

☐ a cat

☐ a dog

working

2. 5 children each own a different pet: a pony, a bird, a fish, a rabbit and a dog. Nina owns a rabbit. Kai owns a bird. Hugo owns a fish. Ivy owns a dog. What does Sam own? C5DKCX

☐ a bird

☐ a rabbit

☐ a pony

☐ a fish

working

3. 5 children each own a different pet: a fish, a cat, a dog, a bird and a pony. Jack owns a bird. Ella owns a dog. Ivy owns a cat. Noah owns a pony. What does Lily own? DF4VTT

☐ a bird

☐ a cat

☐ a dog

☐ a fish

working

4. 5 children each own a different pet: a dog, a cat, a bird, a rabbit and a fish. Ella owns a bird. Zoe owns a cat. Ivy owns a fish. Eli owns a rabbit. What does Max own? L88VCJ

☐ a bird ☐ a cat ☐ a rabbit ☐ a dog

working

5. 5 children each own a different pet: a cat, a bird, a dog, a rabbit and a fish. Finn owns a fish. Nina owns a rabbit. Max owns a cat. Ava owns a dog. What does Owen own? ZFWDGG

☐ a rabbit ☐ a bird ☐ a dog ☐ a cat

working

6. 5 children each own a different pet: a bird, a cat, a pony, a rabbit and a dog. Hugo owns a cat. Mia owns a pony. Lily owns a bird. Mila owns a dog. What does Finn own? QX9ETP

☐ a rabbit ☐ a cat ☐ a pony ☐ a bird

working

7. 5 children each own a different pet: a dog, a cat, a bird, a rabbit and a pony. Finn owns a dog. Nina owns a pony. Mia owns a cat. Ava owns a bird. What does Zoe own? NJ95T3

☐ a rabbit ☐ a cat ☐ a dog ☐ a bird

working

8. 5 children each own a different pet: a fish, a cat, a pony, a dog and a bird. Owen owns a fish. Noah owns a bird. Jack owns a dog. Grace owns a cat. What does Mia own? 5VKK7U

☐ a cat ☐ a dog ☐ a pony ☐ a fish

working

9. 5 children each own a different pet: a rabbit, a cat, a dog, a bird and a pony. Lily owns a pony. Max owns a dog. Hugo owns a cat. Leo owns a rabbit. What does Eli own? WBLWPV

☐ a rabbit ☐ a cat ☐ a dog ☐ a bird

working

10. 5 children each own a different pet: a rabbit, a pony, a dog, a fish and a bird. Ivy owns a bird. Kai owns a rabbit. Leo owns a pony. Noah owns a dog. What does Ella own? ASDPAP

☐ a dog

☐ a rabbit

☐ a pony

☐ a fish

working

11. 5 children each own a different pet: a cat, a dog, a bird, a rabbit and a fish. Ella owns a dog. Jack owns a fish. Ruby owns a rabbit. Lily owns a cat. What does Mila own? WHBUYV

☐ a bird

☐ a cat

☐ a dog

☐ a rabbit

working

12. 5 children each own a different pet: a bird, a rabbit, a pony, a cat and a dog. Ivy owns a rabbit. Jack owns a pony. Noah owns a dog. Sam owns a cat. What does Ruby own? 7AP7AK

☐ a rabbit

☐ a cat

☐ a bird

☐ a pony

working

13. 5 children each own a different pet: a pony, a rabbit, a bird, a fish and a cat. Zoe owns a pony. Sam owns a rabbit. Mila owns a bird. Hugo owns a cat. What does Ruby own? 9QZD27

☐ a rabbit

☐ a bird

☐ a pony

☐ a fish

working

14. 5 children each own a different pet: a bird, a fish, a rabbit, a cat and a dog. Grace owns a fish. Ella owns a bird. Ava owns a cat. Hugo owns a rabbit. What does Leo own? CTZ9ET

☐ a bird

☐ a rabbit

☐ a fish

☐ a dog

working

15. 5 children each own a different pet: a cat, a rabbit, a fish, a pony and a dog. Max owns a pony. Finn owns a fish. Leo owns a dog. Eli owns a cat. What does Zoe own? 3SFGB4

☐ a pony

☐ a fish

☐ a cat

☐ a rabbit

working

16. 5 children each own a different pet: a fish, a dog, a pony, a rabbit and a bird. Noah owns a fish. Ivy owns a bird. Mia owns a dog. Ruby owns a rabbit. What does Finn own?

H2Y6SN

☐ a pony

☐ a fish

☐ a dog

☐ a rabbit

working

17. 5 children each own a different pet: a rabbit, a bird, a cat, a pony and a dog. Zoe owns a dog. Leo owns a cat. Nina owns a pony. Ruby owns a bird. What does Mila own?

HVTDSE

☐ a cat

☐ a bird

☐ a pony

☐ a rabbit

working

18. 5 children each own a different pet: a bird, a pony, a fish, a rabbit and a dog. Eli owns a pony. Finn owns a dog. Jack owns a rabbit. Grace owns a bird. What does Ella own?

QLEB96

☐ a bird

☐ a pony

☐ a rabbit

☐ a fish

working

19. 5 children each own a different pet: a bird, a fish, a dog, a rabbit and a pony. Nina owns a fish. Jack owns a pony. Kai owns a dog. Noah owns a rabbit. What does Hugo own?

QJ9NE3

☐ a dog

☐ a rabbit

☐ a fish

☐ a bird

working

20. 5 children each own a different pet: a rabbit, a dog, a bird, a pony and a cat. Max owns a pony. Kai owns a bird. Leo owns a dog. Ivy owns a cat. What does Ella own?

6798SN

☐ a rabbit

☐ a pony

☐ a bird

☐ a dog

working

21. 5 children each own a different pet: a bird, a cat, a fish, a dog and a rabbit. Nina owns a dog. Ava owns a bird. Leo owns a cat. Ella owns a fish. What does Jack own?

VSWGJA

☐ a cat

☐ a rabbit

☐ a fish

☐ a bird

working

22. 5 children each own a different pet: a cat, a dog, a pony, a bird and a rabbit. Lily owns a rabbit. Ruby owns a bird. Finn owns a pony. Mila owns a dog. What does Noah own?

9XP3FR

☐ a dog

☐ a pony

☐ a bird

☐ a cat

working

23. 5 children each own a different pet: a pony, a cat, a rabbit, a fish and a bird. Ivy owns a pony. Nina owns a cat. Zoe owns a fish. Grace owns a rabbit. What does Mila own?

L5GWJX

☐ a pony

☐ a bird

☐ a cat

☐ a rabbit

working

24. 5 children each own a different pet: a bird, a fish, a rabbit, a pony and a cat. Jack owns a fish. Kai owns a cat. Lily owns a rabbit. Ivy owns a pony. What does Ava own?

HME6PW

☐ a pony

☐ a rabbit

☐ a fish

☐ a bird

working

25. 5 children each own a different pet: a pony, a dog, a cat, a rabbit and a bird. Grace owns a dog. Sam owns a pony. Kai owns a bird. Ava owns a rabbit. What does Mia own?

ZFKAGT

☐ a cat

☐ a rabbit

☐ a dog

☐ a pony

working

26. 5 children each own a different pet: a rabbit, a bird, a pony, a dog and a cat. Lily owns a dog. Kai owns a pony. Ava owns a bird. Mila owns a rabbit. What does Mia own?

T6HFKK

☐ a bird

☐ a rabbit

☐ a cat

☐ a pony

working

27. 5 children each own a different pet: a pony, a bird, a cat, a fish and a dog. Sam owns a fish. Noah owns a bird. Ruby owns a dog. Ava owns a pony. What does Max own?

XLQXVT

☐ a bird

☐ a cat

☐ a fish

☐ a pony

working

28. 5 children each own a different pet: a dog, a pony, a fish, a rabbit and a cat. Ella owns a pony. Sam owns a dog. Nina owns a cat. Ivy owns a rabbit. What does Zoe own?

BHHUD7

☐ a pony

☐ a rabbit

☐ a dog

☐ a fish

working

29. 5 children each own a different pet: a cat, a fish, a bird, a pony and a rabbit. Sam owns a fish. Eli owns a cat. Hugo owns a bird. Nina owns a pony. What does Owen own?

D3Z4BL

☐ a fish

☐ a cat

☐ a bird

☐ a rabbit

working

30. 5 children each own a different pet: a fish, a pony, a bird, a rabbit and a dog. Ivy owns a rabbit. Lily owns a pony. Zoe owns a dog. Hugo owns a fish. What does Grace own?

VXWGP2

☐ a rabbit

☐ a fish

☐ a bird

☐ a pony

working

Chapter 4 — Answers: Ruling things out

1. a fish

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a fish.

2. a pony

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a pony.

3. a fish

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a fish.

4. a dog

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a dog.

5. a bird

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a bird.

6. a rabbit

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a rabbit.

7. a rabbit

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a rabbit.

8. a pony

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a pony.

9. a bird

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a bird.

10. a fish

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a fish.

11. a bird

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a bird.

12. a bird

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a bird.

13. a fish

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a fish.

14. a dog

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a dog.

15. a rabbit

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a rabbit.

16. a pony

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a pony.

17. a rabbit

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a rabbit.

18. a fish

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a fish.

19. a bird

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a bird.

20. a rabbit

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a rabbit.

21. a rabbit

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a rabbit.

22. a cat

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a cat.

23. a bird

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a bird.

24. a bird

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a bird.

25. a cat

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a cat.

26. a cat

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a cat.

27. a cat

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a cat.

28. a fish

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a fish.

29. a rabbit

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a rabbit.

30. a bird

Every other child is named with a pet, and each pet belongs to exactly one child, so the one left over is a bird.

Days a long way out

IN PLAIN WORDS

Days repeat every 7. So counting on 100 days is the same as counting on whatever is left after taking out whole weeks.

EXAMPLE

100 days from Monday. 100 divided by 7 is 14 weeks and 2 left over. Two days on from Monday is Wednesday.

COMMON ERROR

Counting one day at a time and losing your place. Divide by 7 and use the remainder - it is the same answer with none of the risk.

REMEMBER IT LIKE THIS

A clock face with seven hours. Going round it fourteen times changes nothing; only the leftover steps matter.

CHECK YOURSELF

What is the remainder when 30 is divided by 7? (answer: 2)

1. Today is Wednesday. What day will it be in 58 days?

Z3JW4L

☐ Friday

☐ Monday

☐ Saturday

☐ Thursday

working

2. Today is Thursday. What day was it 137 days ago?

2EXUN5

☐ Thursday

☐ Monday

☐ Saturday

☐ Sunday

working

3. Today is Wednesday. What day will it be in 37 days?

FRVYD6

☐ Friday

☐ Saturday

☐ Thursday

☐ Monday

working

4. Today is Wednesday. What day was it 136 days ago?

A78TVD

☐ Monday

☐ Thursday

☐ Sunday

☐ Saturday

working

5. Today is Saturday. What day will it be in 187 days?

GJZT8V

☐ Wednesday ☐ Thursday ☐ Sunday ☐ Friday

working

6. Today is Friday. What day was it 58 days ago?

LYMA9S

☐ Thursday ☐ Sunday ☐ Wednesday ☐ Tuesday

working

7. Today is Friday. What day will it be in 97 days?

7AY8QR

☐ Sunday ☐ Wednesday ☐ Friday ☐ Thursday

working

8. Today is Wednesday. What day was it 129 days ago?

99G5ME

☐ Sunday ☐ Thursday ☐ Monday ☐ Saturday

working

9. Today is Tuesday. What day will it be in 183 days?

CHU55W

☐ Wednesday ☐ Saturday ☐ Tuesday ☐ Thursday

working

10. Today is Tuesday. What day was it 57 days ago?

DG8DRJ

☐ Friday ☐ Tuesday ☐ Monday ☐ Sunday

working

11. Today is Friday. What day will it be in 124 days?

RQV8KT

☐ Tuesday ☐ Wednesday ☐ Thursday ☐ Saturday

working

12. Today is Friday. What day was it 83 days ago?

BURFUW

☐ Friday

☐ Sunday

☐ Wednesday

☐ Saturday

working

13. Today is Tuesday. What day will it be in 121 days?

RW6PEG

☐ Friday

☐ Wednesday

☐ Thursday

☐ Sunday

working

14. Today is Tuesday. What day was it 66 days ago?

NTCDGD

☐ Friday

☐ Sunday

☐ Saturday

☐ Wednesday

working

15. Today is Saturday. What day will it be in 145 days?

A6H8FX

☐ Wednesday

☐ Sunday

☐ Thursday

☐ Friday

working

16. Today is Monday. What day was it 83 days ago?

ADGHJH

☐ Tuesday

☐ Wednesday

☐ Monday

☐ Saturday

working

17. Today is Thursday. What day will it be in 63 days?

MPKQSW

☐ Thursday

☐ Friday

☐ Sunday

☐ Wednesday

working

18. Today is Monday. What day was it 136 days ago?

6E2SJJ

☐ Thursday

☐ Friday

☐ Tuesday

☐ Saturday

working

19. Today is Wednesday. What day will it be in 70 days?

SSAWEL

☐ Wednesday

☐ Saturday

☐ Thursday

☐ Tuesday

working

20. Today is Saturday. What day was it 32 days ago?

BRV2DP

☐ Monday

☐ Saturday

☐ Wednesday

☐ Tuesday

working

21. Today is Saturday. What day will it be in 116 days?

T3SM3T

☐ Tuesday

☐ Thursday

☐ Wednesday

☐ Saturday

working

22. Today is Thursday. What day was it 84 days ago?

DT29VM

☐ Wednesday

☐ Monday

☐ Thursday

☐ Friday

working

23. Today is Saturday. What day will it be in 139 days?

TETRZ7

☐ Thursday

☐ Friday

☐ Monday

☐ Saturday

working

24. Today is Thursday. What day was it 70 days ago?

829ANT

☐ Friday

☐ Thursday

☐ Monday

☐ Wednesday

working

25. Today is Monday. What day will it be in 186 days?

DVXHEZ

☐ Saturday

☐ Monday

☐ Friday

☐ Thursday

working

26. Today is Saturday. What day was it 103 days ago?

VR7NBT

☐ Tuesday ☐ Sunday ☐ Monday ☐ Friday

working

27. Today is Tuesday. What day will it be in 127 days?

NBH8G5

☐ Tuesday ☐ Wednesday ☐ Thursday ☐ Saturday

working

28. Today is Wednesday. What day was it 27 days ago?

FKVGR7

☐ Thursday ☐ Wednesday ☐ Monday ☐ Friday

working

29. Today is Monday. What day will it be in 118 days?

4YPHG5

☐ Wednesday ☐ Monday ☐ Saturday ☐ Sunday

working

30. Today is Saturday. What day was it 37 days ago?

23KBNP

☐ Monday ☐ Thursday ☐ Wednesday ☐ Friday

working

Chapter 5 — Answers: Days a long way out

1. Friday

58 divided by 7 leaves a remainder of 2. Count 2 day(s) on from Wednesday: Friday.

2. Sunday

137 divided by 7 leaves a remainder of 4. Count 4 day(s) back from Thursday: Sunday.

3. Friday

37 divided by 7 leaves a remainder of 2. Count 2 day(s) on from Wednesday: Friday.

4. Sunday

136 divided by 7 leaves a remainder of 3. Count 3 day(s) back from Wednesday: Sunday.

5. Thursday

187 divided by 7 leaves a remainder of 5. Count 5 day(s) on from Saturday: Thursday.

6. Wednesday

58 divided by 7 leaves a remainder of 2. Count 2 day(s) back from Friday: Wednesday.

7. Thursday

97 divided by 7 leaves a remainder of 6. Count 6 day(s) on from Friday: Thursday.

8. Sunday

129 divided by 7 leaves a remainder of 3. Count 3 day(s) back from Wednesday: Sunday.

9. Wednesday

183 divided by 7 leaves a remainder of 1. Count 1 day(s) on from Tuesday: Wednesday.

10. Monday

57 divided by 7 leaves a remainder of 1. Count 1 day(s) back from Tuesday: Monday.

11. Wednesday

124 divided by 7 leaves a remainder of 5. Count 5 day(s) on from Friday: Wednesday.

12. Saturday

83 divided by 7 leaves a remainder of 6. Count 6 day(s) back from Friday: Saturday.

13. Thursday

121 divided by 7 leaves a remainder of 2. Count 2 day(s) on from Tuesday: Thursday.

14. Saturday

66 divided by 7 leaves a remainder of 3. Count 3 day(s) back from Tuesday: Saturday.

15. Thursday

145 divided by 7 leaves a remainder of 5. Count 5 day(s) on from Saturday: Thursday.

16. Tuesday

83 divided by 7 leaves a remainder of 6. Count 6 day(s) back from Monday: Tuesday.

17. Thursday

63 divided by 7 leaves a remainder of 0. Count 0 day(s) on from Thursday: Thursday.

18. Friday

136 divided by 7 leaves a remainder of 3. Count 3 day(s) back from Monday: Friday.

19. Wednesday

70 divided by 7 leaves a remainder of 0. Count 0 day(s) on from Wednesday: Wednesday.

20. Tuesday

32 divided by 7 leaves a remainder of 4. Count 4 day(s) back from Saturday: Tuesday.

21. Wednesday

116 divided by 7 leaves a remainder of 4. Count 4 day(s) on from Saturday: Wednesday.

22. Thursday

84 divided by 7 leaves a remainder of 0. Count 0 day(s) back from Thursday: Thursday.

23. Friday

139 divided by 7 leaves a remainder of 6. Count 6 day(s) on from Saturday: Friday.

24. Thursday

70 divided by 7 leaves a remainder of 0. Count 0 day(s) back from Thursday: Thursday.

25. Friday

186 divided by 7 leaves a remainder of 4. Count 4 day(s) on from Monday: Friday.

26. Monday

103 divided by 7 leaves a remainder of 5. Count 5 day(s) back from Saturday: Monday.

27. Wednesday

127 divided by 7 leaves a remainder of 1. Count 1 day(s) on from Tuesday: Wednesday.

28. Thursday

27 divided by 7 leaves a remainder of 6. Count 6 day(s) back from Wednesday: Thursday.

29. Sunday

118 divided by 7 leaves a remainder of 6. Count 6 day(s) on from Monday: Sunday.

30. Thursday

37 divided by 7 leaves a remainder of 2. Count 2 day(s) back from Saturday: Thursday.

Balancing three things

IN PLAIN WORDS

If a cube is worth some spheres, and a sphere is worth some cones, then a cube is worth the two numbers multiplied together. Convert down to the smallest thing and compare there.

EXAMPLE

One cube is 3 spheres, one sphere is 2 cones. So one cube is $3 \times 2 = 6$ cones.

COMMON ERROR

Adding the two numbers. Three spheres and two cones does not make five of anything - the units are different at each step.

REMEMBER IT LIKE THIS

Dollars to cents. You multiply going down and divide coming back up.

CHECK YOURSELF

One block is 4 cones. How many cones balance 3 blocks? (answer: 12)

1. One cube balances 4 spheres. One sphere balances 2 cones. How many cones balance 2 cubes?

TGL9HJ

☐ 6☐ 4☐ 16☐ 8

working

2. One block balances 4 cones. How many blocks balance 32 cones?

DTXXWF

☐ 28☐ 128☐ 4☐ 8

working

3. One cube balances 2 spheres. One sphere balances 5 cones. How many cones balance 4 cubes?

3UD75E

☐ 8☐ 20☐ 40☐ 7

working

4. One block balances 5 cones. How many blocks balance 40 cones?

XWCV5H

☐ 5☐ 35☐ 8☐ 200

working

5. One cube balances 4 spheres. One sphere balances 2 cones. How many cones balance 5 cubes?

9GCYE2

☐ 40

☐ 10

☐ 6

☐ 20

working

6. One block balances 7 cones. How many blocks balance 63 cones?

JXRF4V

☐ 441

☐ 7

☐ 56

☐ 9

working

7. One cube balances 4 spheres. One sphere balances 3 cones. How many cones balance 6 cubes?

QB8PGF

☐ 7

☐ 24

☐ 72

☐ 18

working

8. One block balances 2 cones. How many blocks balance 12 cones?

PQY9T2

☐ 24

☐ 2

☐ 6

☐ 10

working

9. One cube balances 2 spheres. One sphere balances 5 cones. How many cones balance 2 cubes?

H3D939

☐ 10

☐ 7

☐ 20

☐ 4

working

10. One block balances 7 cones. How many blocks balance 56 cones?

HHX9HD

☐ 8

☐ 49

☐ 392

☐ 7

working

11. One cube balances 4 spheres. One sphere balances 3 cones. How many cones balance 5 cubes?

NC65JV

☐ 20

☐ 60

☐ 15

☐ 7

working

12. One block balances 7 cones. How many blocks balance 21 cones?

XM6YWK

☐ 147

☐ 7

☐ 14

☐ 3

working

13. One cube balances 3 spheres. One sphere balances 5 cones. How many cones balance 4 cubes?

QTT5V4

☐ 60

☐ 20

☐ 12

☐ 8

working

14. One block balances 7 cones. How many blocks balance 35 cones?

BV8C8A

☐ 5

☐ 7

☐ 245

☐ 28

working

15. One cube balances 4 spheres. One sphere balances 5 cones. How many cones balance 5 cubes?

JCBMZP

☐ 20

☐ 25

☐ 100

☐ 9

working

16. One block balances 2 cones. How many blocks balance 14 cones?

DVKUZD

☐ 2

☐ 12

☐ 28

☐ 7

working

17. One cube balances 4 spheres. One sphere balances 2 cones. How many cones balance 6 cubes?

CTNAMM

☐ 48

☐ 12

☐ 24

☐ 6

working

18. One block balances 6 cones. How many blocks balance 48 cones?

K3N6BC

☐ 42

☐ 8

☐ 288

☐ 6

working

19. One cube balances 2 spheres. One sphere balances 3 cones. How many cones balance 3 cubes?

ZU3RFU

☐ 9

☐ 18

☐ 6

☐ 5

working

20. One block balances 7 cones. How many blocks balance 42 cones?

8NCYBY

☐ 6

☐ 35

☐ 294

☐ 7

working

21. One cube balances 5 spheres. One sphere balances 4 cones. How many cones balance 3 cubes?

QE9C9U

☐ 9

☐ 60

☐ 12

☐ 15

working

22. One block balances 4 cones. How many blocks balance 24 cones?

HFJVEQ

☐ 6

☐ 4

☐ 20

☐ 96

working

23. One cube balances 3 spheres. One sphere balances 4 cones. How many cones balance 4 cubes?

4ECEBW

☐ 16

☐ 48

☐ 7

☐ 12

working

24. One block balances 2 cones. How many blocks balance 10 cones?

PTM76W

☐ 8

☐ 2

☐ 5

☐ 20

working

25. One cube balances 3 spheres. One sphere balances 4 cones. How many cones balance 6 cubes?

CT7N3V

☐ 72

☐ 7

☐ 18

☐ 24

working

26. One block balances 5 cones. How many blocks balance 45 cones?

CJKE28

☐ 5

☐ 40

☐ 9

☐ 225

working

27. One cube balances 2 spheres. One sphere balances 5 cones. How many cones balance 5 cubes?

RPSDNB

☐ 25

☐ 50

☐ 10

☐ 7

working

28. One block balances 5 cones. How many blocks balance 30 cones?

S6GCLJ

☐ 25

☐ 5

☐ 6

☐ 150

working

29. One cube balances 5 spheres. One sphere balances 3 cones. How many cones balance 3 cubes?

7ATNHX

☐ 15

☐ 9

☐ 45

☐ 8

working

30. One block balances 3 cones. How many blocks balance 27 cones?

3WREAV

☐ 9

☐ 24

☐ 81

☐ 3

working

Chapter 6 — Answers: Balancing three things

1. 16

One cube is 4 spheres, and each sphere is 2 cones, so one cube is $4 \times 2 = 8$ cones. 2 cubes is $2 \times 8 = 16$.

2. 8

Each block is worth 4 cones, so $32 / 4 = 8$ blocks.

3. 40

One cube is 2 spheres, and each sphere is 5 cones, so one cube is $2 \times 5 = 10$ cones. 4 cubes is $4 \times 10 = 40$.

4. 8

Each block is worth 5 cones, so $40 / 5 = 8$ blocks.

5. 40

One cube is 4 spheres, and each sphere is 2 cones, so one cube is $4 \times 2 = 8$ cones. 5 cubes is $5 \times 8 = 40$.

6. 9

Each block is worth 7 cones, so $63 / 7 = 9$ blocks.

7. 72

One cube is 4 spheres, and each sphere is 3 cones, so one cube is $4 \times 3 = 12$ cones. 6 cubes is $6 \times 12 = 72$.

8. 6

Each block is worth 2 cones, so $12 / 2 = 6$ blocks.

9. 20

One cube is 2 spheres, and each sphere is 5 cones, so one cube is $2 \times 5 = 10$ cones. 2 cubes is $2 \times 10 = 20$.

10. 8

Each block is worth 7 cones, so $56 / 7 = 8$ blocks.

11. 60

One cube is 4 spheres, and each sphere is 3 cones, so one cube is $4 \times 3 = 12$ cones. 5 cubes is $5 \times 12 = 60$.

12. 3

Each block is worth 7 cones, so $21 / 7 = 3$ blocks.

13. 60

One cube is 3 spheres, and each sphere is 5 cones, so one cube is $3 \times 5 = 15$ cones. 4 cubes is $4 \times 15 = 60$.

14. 5

Each block is worth 7 cones, so $35 / 7 = 5$ blocks.

15. 100

One cube is 4 spheres, and each sphere is 5 cones, so one cube is $4 \times 5 = 20$ cones. 5 cubes is $5 \times 20 = 100$.

16. 7

Each block is worth 2 cones, so $14 / 2 = 7$ blocks.

17. 48

One cube is 4 spheres, and each sphere is 2 cones, so one cube is $4 \times 2 = 8$ cones. 6 cubes is $6 \times 8 = 48$.

18. 8

Each block is worth 6 cones, so $48 / 6 = 8$ blocks.

19. 18

One cube is 2 spheres, and each sphere is 3 cones, so one cube is $2 \times 3 = 6$ cones. 3 cubes is $3 \times 6 = 18$.

20. 6

Each block is worth 7 cones, so $42 / 7 = 6$ blocks.

21. 60

One cube is 5 spheres, and each sphere is 4 cones, so one cube is $5 \times 4 = 20$ cones. 3 cubes is $3 \times 20 = 60$.

22. 6

Each block is worth 4 cones, so $24 / 4 = 6$ blocks.

23. 48

One cube is 3 spheres, and each sphere is 4 cones, so one cube is $3 \times 4 = 12$ cones. 4 cubes is $4 \times 12 = 48$.

24. 5

Each block is worth 2 cones, so $10 / 2 = 5$ blocks.

25. 72

One cube is 3 spheres, and each sphere is 4 cones, so one cube is $3 \times 4 = 12$ cones. 6 cubes is $6 \times 12 = 72$.

26. 9

Each block is worth 5 cones, so $45 / 5 = 9$ blocks.

27. 50

One cube is 2 spheres, and each sphere is 5 cones, so one cube is $2 \times 5 = 10$ cones. 5 cubes is $5 \times 10 = 50$.

28. 6

Each block is worth 5 cones, so $30 / 5 = 6$ blocks.

29. 45

One cube is 5 spheres, and each sphere is 3 cones, so one cube is $5 \times 3 = 15$ cones. 3 cubes is $3 \times 15 = 45$.

30. 9

Each block is worth 3 cones, so $27 / 3 = 9$ blocks.

Machines that do two things

IN PLAIN WORDS

Work out the rule from the examples you are given, then check it against every example before you use it.

EXAMPLE

1 goes to 5, 2 goes to 8, 3 goes to 11. The jumps are 3 each time and the rule is times 3 add 2. Check: $2 \times 3 + 2 = 8$. It holds.

COMMON ERROR

Finding a rule that fits the first example and using it straight away. Times 5 also turns 1 into 5, and then fails on everything else.

REMEMBER IT LIKE THIS

A key that turns in the first lock is not the right key if it jams in the second.

CHECK YOURSELF

1 goes to 7, 2 goes to 12, 3 goes to 17. What is the rule? (answer: times 5, add 2)

1. A machine turns 1 into 12, 2 into 15, and 3 into 18. What does it turn 2 into?

DJBYGH

☐ 11

☐ 15

☐ 24

☐ 6

working

2. A machine multiplies by 2 and then adds 5. It has just produced 11. What went in?

ZCTZNE

☐ 3

☐ 5

☐ 6

☐ 11

working

3. A machine turns 1 into 11, 2 into 13, and 3 into 15. What does it turn 5 into?

QUB2M6

☐ 55

☐ 19

☐ 14

☐ 10

working

4. A machine multiplies by 5 and then adds 5. It has just produced 25. What went in?

7DMDHM

☐ 5

☐ 4

☐ 25

☐ 20

working

5. A machine turns 1 into 4, 2 into 6, and 3 into 8. What does it turn 12 into?

J9C4HR

☐ 14

☐ 24

☐ 26

☐ 48

working

6. A machine multiplies by 5 and then adds 7. It has just produced 22. What went in?

UEQUT4

☐ 3

☐ 22

☐ 4

☐ 15

working

7. A machine turns 1 into 4, 2 into 6, and 3 into 8. What does it turn 8 into?

9LZTEH

☐ 32

☐ 10

☐ 18

☐ 16

working

8. A machine multiplies by 5 and then adds 6. It has just produced 71. What went in?

9N7TDJ

☐ 71

☐ 13

☐ 14

☐ 65

working

9. A machine turns 1 into 10, 2 into 13, and 3 into 16. What does it turn 4 into?

36A9SS

☐ 11

☐ 12

☐ 40

☐ 19

working

10. A machine multiplies by 3 and then adds 9. It has just produced 21. What went in?

3RMN8T

☐ 7

☐ 21

☐ 4

☐ 12

working

11. A machine turns 1 into 10, 2 into 12, and 3 into 14. What does it turn 10 into?

ANKQZJ

☐ 28

☐ 18

☐ 100

☐ 20

working

12. A machine multiplies by 5 and then adds 5. It has just produced 40. What went in?

P72HF4

☐ 35

☐ 8

☐ 40

☐ 7

working

13. A machine turns 1 into 8, 2 into 12, and 3 into 16. What does it turn 5 into?

R5QPUK

☐ 20

☐ 24

☐ 9

☐ 40

working

14. A machine multiplies by 5 and then adds 10. It has just produced 65. What went in?

W4V8T5

☐ 55

☐ 65

☐ 11

☐ 13

working

15. A machine turns 1 into 7, 2 into 10, and 3 into 13. What does it turn 5 into?

SGSFCG

☐ 15

☐ 35

☐ 9

☐ 19

working

16. A machine multiplies by 5 and then adds 5. It has just produced 20. What went in?

R2HLDR

☐ 15

☐ 4

☐ 20

☐ 3

working

17. A machine turns 1 into 11, 2 into 15, and 3 into 19. What does it turn 6 into?

9PWLTD

☐ 31

☐ 66

☐ 13

☐ 24

working

18. A machine multiplies by 2 and then adds 9. It has just produced 39. What went in?

EWQCW9

☐ 19

☐ 30

☐ 39

☐ 15

working

19. A machine turns 1 into 7, 2 into 9, and 3 into 11. What does it turn 8 into?

7NJ633

☐ 13

☐ 56

☐ 21

☐ 16

working

20. A machine multiplies by 2 and then adds 9. It has just produced 27. What went in?

Y6HF8D

☐ 13

☐ 27

☐ 9

☐ 18

working

21. A machine turns 1 into 7, 2 into 11, and 3 into 15. What does it turn 5 into?

L4LTFQ

☐ 23

☐ 35

☐ 20

☐ 8

working

22. A machine multiplies by 2 and then adds 6. It has just produced 18. What went in?

7HHDGD

☐ 12

☐ 18

☐ 9

☐ 6

working

23. A machine turns 1 into 7, 2 into 10, and 3 into 13. What does it turn 4 into?

VMJGYT

☐ 16

☐ 8

☐ 28

☐ 12

working

24. A machine multiplies by 2 and then adds 2. It has just produced 8. What went in?

487RXK

☐ 6

☐ 8

☐ 4

☐ 3

working

25. A machine turns 1 into 7, 2 into 11, and 3 into 15. What does it turn 8 into?

N7LDRV

☐ 35

☐ 56

☐ 32

☐ 11

working

26. A machine multiplies by 3 and then adds 10. It has just produced 49. What went in?

YXR37X

☐ 49

☐ 13

☐ 39

☐ 16

working

27. A machine turns 1 into 9, 2 into 11, and 3 into 13. What does it turn 11 into?

72DBAV

☐ 18

☐ 99

☐ 29

☐ 22

working

28. A machine multiplies by 2 and then adds 4. It has just produced 26. What went in?

UN2C9G

☐ 13

☐ 26

☐ 11

☐ 22

working

29. A machine turns 1 into 11, 2 into 15, and 3 into 19. What does it turn 4 into?

6XAERY

☐ 44

☐ 16

☐ 11

☐ 23

working

30. A machine multiplies by 2 and then adds 2. It has just produced 32. What went in?

84D9HB

☐ 16

☐ 30

☐ 15

☐ 32

working

Chapter 7 — Answers: Machines that do two things

1. 15

Each output is 3 times the input plus 9. So 2 gives $2 \times 3 + 9 = 15$.

2. 3

Undo the add first: $11 - 5 = 6$. Then undo the multiply: $6 \div 2 = 3$.

3. 19

Each output is 2 times the input plus 9. So 5 gives $5 \times 2 + 9 = 19$.

4. 4

Undo the add first: $25 - 5 = 20$. Then undo the multiply: $20 \div 5 = 4$.

5. 26

Each output is 2 times the input plus 2. So 12 gives $12 \times 2 + 2 = 26$.

6. 3

Undo the add first: $22 - 7 = 15$. Then undo the multiply: $15 \div 5 = 3$.

7. 18

Each output is 2 times the input plus 2. So 8 gives $8 \times 2 + 2 = 18$.

8. 13

Undo the add first: $71 - 6 = 65$. Then undo the multiply: $65 \div 5 = 13$.

9. 19

Each output is 3 times the input plus 7. So 4 gives $4 \times 3 + 7 = 19$.

10. 4

Undo the add first: $21 - 9 = 12$. Then undo the multiply: $12 \div 3 = 4$.

11. 28

Each output is 2 times the input plus 8. So 10 gives $10 \times 2 + 8 = 28$.

12. 7

Undo the add first: $40 - 5 = 35$. Then undo the multiply: $35 \div 5 = 7$.

13. 24

Each output is 4 times the input plus 4. So 5 gives $5 \times 4 + 4 = 24$.

14. 11

Undo the add first: $65 - 10 = 55$. Then undo the multiply: $55 \div 5 = 11$.

15. 19

Each output is 3 times the input plus 4. So 5 gives $5 \times 3 + 4 = 19$.

16. 3

Undo the add first: $20 - 5 = 15$. Then undo the multiply: $15 \div 5 = 3$.

17. 31

Each output is 4 times the input plus 7. So 6 gives $6 \times 4 + 7 = 31$.

18. 15

Undo the add first: $39 - 9 = 30$. Then undo the multiply: $30 \div 2 = 15$.

19. 21

Each output is 2 times the input plus 5. So 8 gives $8 \times 2 + 5 = 21$.

20. 9

Undo the add first: $27 - 9 = 18$. Then undo the multiply: $18 \div 2 = 9$.

21. 23

Each output is 4 times the input plus 3. So 5 gives $5 \times 4 + 3 = 23$.

22. 6

Undo the add first: $18 - 6 = 12$. Then undo the multiply: $12 / 2 = 6$.

23. 16

Each output is 3 times the input plus 4. So 4 gives $4 \times 3 + 4 = 16$.

24. 3

Undo the add first: $8 - 2 = 6$. Then undo the multiply: $6 / 2 = 3$.

25. 35

Each output is 4 times the input plus 3. So 8 gives $8 \times 4 + 3 = 35$.

26. 13

Undo the add first: $49 - 10 = 39$. Then undo the multiply: $39 / 3 = 13$.

27. 29

Each output is 2 times the input plus 7. So 11 gives $11 \times 2 + 7 = 29$.

28. 11

Undo the add first: $26 - 4 = 22$. Then undo the multiply: $22 / 2 = 11$.

29. 23

Each output is 4 times the input plus 7. So 4 gives $4 \times 4 + 7 = 23$.

30. 15

Undo the add first: $32 - 2 = 30$. Then undo the multiply: $30 / 2 = 15$.